

# Scheme for implementing the Deutsch-Jozsa algorithm via atomic ensembles

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We propose a physical scheme for implementing the Deutsch-Jozsa algorithm using atomic ensembles and optical devices. The scheme has inherent fault tolerance to the realistic noise and efficient scaling with the number of ensembles for some entangled states within the reach of current technology. It would be an important step toward more complex quantum computation via atomic ensembles.

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## I. INTRODUCTION

Quantum computation is an enormously appealing task for mathematicians, physical scientists and computer scientists because of its potential to perform superfast quantum algorithms. Quantum factoring [1] and quantum search [2] illustrate the great theoretical promise of quantum computers. Quantum entanglement is a striking feature of quantum mechanics, which can be employed as a kind of “quantum software” to perform quantum computation [3]. A great deal of schemes for generating entangled states have been proposed using cavity quantum electrodynamics (QED) [4, 5, 6], linear optics [7] and so on [8, 9, 10]. As for quantum computation, single-qubit manipulations and C-NOT gates together can be served to realize any unitary operation on  $n$  qubits. The C-NOT gate can be realized via many techniques [11, 12, 13].

The Deutsch-Jozsa algorithm is a simple example of general quantum algorithms, which can distinguish the function  $f(x)$  between constant and balanced [14] on  $2^n$  inputs  $x$ . The values of the function  $f(x)$  are either 0 or 1 for all possible inputs. For the balanced function, the values of balanced function are equal to 1 for half of all the possible inputs, and 0 for the other half. The Deutsch-Jozsa algorithm has been realized in the nuclear magnetic resonance system (NMR) [15], ion trap [16], linear optical system [17], cavity QED [18] and atomic ensembles [19] theoretically and experimentally.

Recently, many researchers have been paying their attentions to atomic ensembles where the basic system is a large number of identical atoms. A lot of interesting schemes for the generation of quantum states and quantum information processing have been proposed using atomic ensembles. For example, one can use atomic ensembles to realize the scalable long-distance quantum communication [20], efficient generation of multipartite entanglement states [21, 22], storage of quantum light [23, 24]. The schemes based on atomic ensembles have some special advantages compared with the schemes of

quantum information processing by the control of single particles: (1) The schemes have inherent fault tolerance and are robust to realistic noise and imperfections; (2) Laser manipulation of atomic ensembles without separately addressing the individual atoms is dominant easier than the coherent control of single particles; (3) Atomic ensembles with suitable level structure could have some kinds of collectively enhanced coupling to certain optical mode due to the many-atom interference effects, which is very important for all the recent schemes based on the atomic ensembles. Here we suggest the implementation of the Deutsch-Jozsa algorithm via atomic ensembles. But our scheme is different from Ref. [19] due to we chose different atomic configuration and use different model. We utilize the collective enhancement of atom ensembles other than considerate the interaction of atoms and photon. Our scheme involves Raman-type laser manipulations, beam splitters, and single-photon detections, the requirements of which are well within the current experimental technology. Obviously the realization of Hadamard gate, C-NOT gate and equivalent Bell-basis measurement is necessary to our ultimate aim. Therefore we will introduce the scheme of these important implements in section II. In section III, the process of implementing the Deutsch-Jozsa algorithm is proposed in detail. The conclusions of the whole content are given in the end section.

## II. REALIZATION OF C-NOT GATE

It has been shown that C-NOT gate can be realized theoretically with the help of GHZ states, Bell-basis measures and Hadamard operations from the proposal in Ref [3]. In this section, we will realize C-NOT gate using atomic ensembles with a large number of identical alkali metal atoms as basic system. The scheme involves Raman-type laser manipulations, beam splitters, and single-photon detections. The relevant level structure of the alkali metal atoms is shown in Fig. 1. For the three levels  $|g\rangle$ ,  $|h\rangle$  and  $|v\rangle$ , two collective atomic operators can be defined as  $S = (1/\sqrt{N})\sum_{i=1}^{N_a}|g\rangle_i\langle s|$ , where  $s = h, v$ , and  $N_a \gg 1$  is the total number of atoms. The three levels  $|g\rangle$ ,  $|h\rangle$  and  $|v\rangle$  can be coupled via a Raman process. The atoms are initially prepared in the ground state  $|g\rangle$

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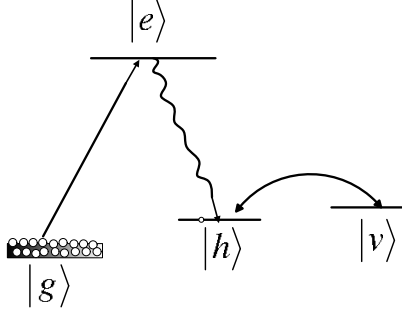


FIG. 1: The relevant atomic level structure of alkali metal atom.  $|g\rangle$  is the ground state,  $|e\rangle$  is the excited state and  $|h\rangle$ ,  $|v\rangle$  are two metastable states (Zeeman or hyperfine sublevels) for storing a qubit of information. The transition of  $|e\rangle \rightarrow |h\rangle$  can emit a forward-scattered Stokes photon that is co-propagating with the laser pulse. The excitation in the mode  $h$  can be transferred to optical excitation by applying an anti-pump pulse.

using optical pump.  $S$  is similar to independent bosonic mode operators if only the atoms are all remain in ground state  $|g\rangle$ . The states of the atomic ensemble can be express as  $|S\rangle = s^+|vac\rangle$  ( $s = h, v$ ) after the emission of the single Stokes photon in a forward direction, where  $|vac\rangle$  denotes the ground state of atomic ensembles and  $|vac\rangle \equiv \otimes_i |g\rangle_i$ . Long time coherence has been demonstrated experimentally both in a room-temperature dilute atomic gas [25] and in a sample of cold trapped atoms [24]. The above character of atoms is useful for the generation of entanglement between atomic ensembles [21, 22]. Single-qubit operations with high precision between the two atomic states  $|S\rangle = s^+|vac\rangle$  ( $s = h, v$ ) can be completed by simply shining Raman pulses or radio-frequency pulses on all the atoms. For instance, we can obtain  $h^+|vac\rangle \rightarrow (h^+ + v^+)|vac\rangle/\sqrt{2}$  and  $v^+|vac\rangle \rightarrow (h^+ - v^+)|vac\rangle/\sqrt{2}$  by choosing appropriate length of the pump and anti-pump pulse. It is equivalent to realize a Hadamard gate, which is very useful in the below scheme.

Bell-basis measurement is very important in the process of realizing C-NOT gate, quantum information processing and quantum computation. Therefore it is necessary to introduce the realization of the Bell-basis measurement based on atomic ensembles in our paper. The four Bell states in the system are  $|\phi\rangle_{AB}^\pm = (h_A^+ h_B^+ \pm v_A^+ v_B^+)|vac\rangle_{AB}/\sqrt{2}$  and  $|\varphi\rangle_{AB}^\pm = (h_A^+ v_B^+ \pm v_A^+ h_B^+)|vac\rangle_{AB}/\sqrt{2}$ . We can use the setup to achieve the task, as shown in Fig. 2. Firstly, we apply anti-pump laser pulses to the two atomic ensembles  $A$  and  $B$  to transfer their  $h$  excitations to optical excitations, and detect the anti-Stokes photons by detectors  $D1$  and  $D2$ . In the case of only detector  $D1$  (or  $D2$ ) clicks. We will apply single-qubit rotations to both ensembles to rotate their  $v$  modes to  $h$  modes by shining  $\pi$  length Raman pulses or radio-frequency pulses on the two ensembles  $A$  and  $B$ . Then we apply anti-pump laser pulses to the two atomic ensembles  $A$  and  $B$  again, and detect anti-

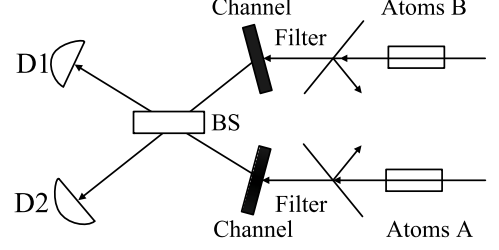


FIG. 2: Setup of realizing Bell-basis measurement. The two atomic ensembles  $A$  and  $B$  are pencil-shaped, which are illuminated by the synchronized laser pulses. The forward-scattered anti-Stokes are collected and coupled to optical channel (fiber) after the filter. BS is a 50/50 beam splitter, and the outputs detected by two single-photon detectors  $D1$  and  $D2$ .

Stokes photons by  $D1$  and  $D2$ . Now, there are two different results of detection: (1) If detector  $D1$  (or  $D2$ ) clicks (i.e. only one detector clicks in the two detections), post-select the cases that each ensemble has only one excitation, atomic ensembles  $A$  and  $B$  are projected to  $|\varphi\rangle_{AB}^+ = (h_A^+ v_B^+ + v_A^+ h_B^+)|vac\rangle_{AB}/\sqrt{2}$ ; (2) If  $D2$  (or  $D1$ ) clicks (i.e. detectors  $D1$  and  $D2$  click respectively in the two detections), post-select the cases that each ensemble has only one excitation, atomic ensembles  $A$  and  $B$  are projected to  $|\varphi\rangle_{AB}^- = (h_A^+ v_B^+ - v_A^+ h_B^+)|vac\rangle_{AB}/\sqrt{2}$ . Obviously, if we add single-qubit rotations in the above process and repeat the above process of (1), we can realize the projection of  $|\phi\rangle_{AB}^\pm = (h_A^+ h_B^+ \pm v_A^+ v_B^+)|vac\rangle_{AB}/\sqrt{2}$  by post-selecting sense.

In order to realize C-NOT gate, we prepare two GHZ states, which have been made in Ref [21] with atomic ensembles 1, 2, 3, 4, 5 and 6

$$|\phi\rangle_{123} = (h_1^+ h_2^+ h_3^+ + v_1^+ v_2^+ v_3^+)|vac\rangle_{123}/\sqrt{2}, \quad (1a)$$

$$|\varphi\rangle_{456} = (h_4^+ h_5^+ h_6^+ + v_4^+ v_5^+ v_6^+)|vac\rangle_{456}/\sqrt{2}, \quad (1b)$$

and prepare two atomic ensembles 7 and 8, which are in  $|\phi\rangle_7 = (h_7^+ + v_7^+)|vac\rangle_7$  and  $|\phi\rangle_8 = (h_8^+ - v_8^+)|vac\rangle_8$  by single-qubit rotations. The C-NOT gate has  $|\phi\rangle_7$  as its control, and  $|\phi\rangle_8$  as its target. At first, we apply Hadamard transformations on atomic ensembles 1, 2 and 3 by single-qubit operations, respectively, and then make a Bell-basis measurement on atomic ensembles 3 and 4 using the setup in Fig. 2. Here, the state of the  $|\varphi\rangle_{123456}$  collapses to one of the following four unnormalized states

$$|\phi\rangle_{1256} = [(h_1^+ h_2^+ + v_1^+ v_2^+)h_5^+ h_6^+ \pm (h_1^+ v_2^+ + v_1^+ h_2^+)v_5^+ v_6^+]|vac\rangle_{1256}, \quad (2a)$$

$$|\varphi\rangle_{1256} = [(h_1^+ h_2^+ + v_1^+ v_2^+)v_5^+ v_6^+ \pm (h_1^+ v_2^+ + v_1^+ h_2^+)h_5^+ h_6^+]|vac\rangle_{1256}. \quad (2b)$$

$|\phi\rangle_{1256}$  and  $|\varphi\rangle_{1256}$  are the results of projecting to  $|\phi\rangle_{34}^{\pm}$  and  $|\varphi\rangle_{34}^{\pm}$ , respectively. They can unify as  $|\chi\rangle_{1256} = [(h_1^+ h_2^+ + v_1^+ v_2^+) h_5^+ h_6^+ + (h_1^+ v_2^+ + v_1^+ h_2^+) v_5^+ v_6^+] |vac\rangle_{1256}$  with the help of simple single-qubit operations.

In succession, we make a Bell-basis measurement on atomic ensembles 1 and 8 as the above techniques. The state of atomic collective 1, 2, 5, 6 and 8 collapses to one of the following two states

$$|\phi\rangle_{256} = [(h_2^+ h_5^+ h_6^+ + v_2^+ v_5^+ v_6^+) + (v_2^+ h_5^+ h_6^+ + h_2^+ v_5^+ v_6^+)] |vac\rangle_{256}, \quad (3a)$$

$$|\varphi\rangle_{256} = [(h_2^+ h_5^+ h_6^+ + v_2^+ v_5^+ v_6^+) - (v_2^+ h_5^+ h_6^+ + h_2^+ v_5^+ v_6^+)] |vac\rangle_{256}, \quad (3b)$$

where Eq. (3a) is the result of projecting to  $|\phi\rangle_{18}^+$  and  $|\varphi\rangle_{18}^+$ , and Eq. (3b) is the result of projecting to  $|\phi\rangle_{18}^-$  and  $|\varphi\rangle_{18}^-$ . Apply single rotations to Eq. (3b), leads the state of atomic collective 2, 5 and 6 to Eq. (3a). We make a Bell-basis measurement on atomic ensembles 6 and 7, the state of atomic collective 2 and 5 collapses to the one of the following states

$$|\phi\rangle_{25} = (h_2^+ - v_2^+)(h_5^+ - v_5^+) |vac\rangle_{25}/2, \quad (4a)$$

$$|\varphi\rangle_{25} = (h_2^+ - v_2^+)(h_5^+ + v_5^+) |vac\rangle_{25}/2. \quad (4b)$$

where Eq. (4a) corresponds to the measurement results of  $|\phi\rangle_{67}^+$  and  $|\varphi\rangle_{67}^+$ , and Eq. (4b) corresponds to  $|\phi\rangle_{67}^-$  and  $|\varphi\rangle_{67}^-$ . We can make state (4b) transform to state (4a) by single-qubit rotations. Obviously here, the C-NOT gate has been realized and the state have mapped on atomic collective 2 and 5. Therefore if only we apply Hadamard transformations, Bell-basis measurements and single rotations on atomic ensembles, we are able to realize C-NOT gate perfectly.

### III. REALIZATION OF THE DEUTSCH-JOZSA ALGORITHM

The Realization of Deutsch-Jozsa algorithm is an important step toward more complex quantum computation. Classically, if we want to distinguish  $f(x)$  between constant and balanced function on  $2^n$  inputs, we will need  $2^n/2 + 1$  queries to unambiguously determine whether the function is balanced. While for the Deutsch-Jozsa algorithm, we will need only one query. Here, we consider the two-qubit Deutsch-Jozsa algorithm. The input query qubit is prepared in  $(|0\rangle_i + |1\rangle_i)/\sqrt{2}$  and the auxiliary working qubit is prepared in  $(|0\rangle_a - |1\rangle_a)/\sqrt{2}$ , so the state of the whole system is  $(|0\rangle_i + |1\rangle_i)(|0\rangle_a - |1\rangle_a)/2$ . While the function  $f(x)$  is characterized by unitary mapping transformation  $U_f$ , and  $|x, y\rangle \rightarrow |x, y \oplus f(x)\rangle$  where  $\oplus$  indicates addition modulo 2. The unitary transformation  $U_f$  on the system leads the initially state to

$$[(-1)^{f(0)}|0\rangle_i + (-1)^{f(1)}|1\rangle_i](|0\rangle_a - |1\rangle_a)/2. \quad (5)$$

There are four possible transformations to the  $U_f$ :

1. for  $U_{f1}$ ,  $f(0) = f(1) = 0$ ;
2. for  $U_{f2}$ ,  $f(0) = f(1) = 1$ ;
3. for  $U_{f3}$ ,  $f(0) = 0$  and  $f(1) = 1$ ;
4. for  $U_{f4}$ ,  $f(0) = 1$  and  $f(1) = 0$ .

After a Hadamard transformation on the input query qubit, the state of query qubit becomes  $|f(0) \oplus f(1)\rangle$ . If the function  $f(x)$  is constant, the state of query qubit becomes  $|0\rangle_i$ . Otherwise it becomes  $|1\rangle_i$ .

Now, we realize the two-qubit Deutsch-Jozsa algorithm via atomic ensembles and linear optical elements. At first, we prepare two atomic ensembles 9 and 10, which are initially in the states  $|\phi\rangle_9 = h_9^+ |vac\rangle_9$  and  $|\phi\rangle_{10} = h_{10}^+ |vac\rangle_{10}$ , respectively. After two single-qubit operations by controlling Raman pulses with the appropriate length, we can obtain the state of the atomic collective 9 and 10 is

$$|\phi\rangle_{910} = (h_9^+ + v_9^+)(h_{10}^+ - v_{10}^+) |vac\rangle_{910}/2, \quad (6)$$

which can be rewritten as

$$|\phi\rangle_{910} = (|0\rangle_9 + |1\rangle_9)(|0\rangle_{10} - |1\rangle_{10})/2, \quad (7)$$

where  $|0\rangle_9 = h_9^+ |vac\rangle_9$ ,  $|1\rangle_9 = v_9^+ |vac\rangle_9$ ,  $|0\rangle_{10} = h_{10}^+ |vac\rangle_{10}$  and  $|1\rangle_{10} = v_{10}^+ |vac\rangle_{10}$ . So the collective state has the same form as that of input and auxiliary working qubit in the Deutsch-Jozsa algorithm.

For the case of performing  $U_{f1}$ , we take no operation on the system and the state remains in the state  $|\phi\rangle_{910}$ .

For the case of performing  $U_{f2}$ , we can make a C-NOT transformation on the two collective 9 and 10 (collective 9 as control bit, collective 10 as target bit), which can be achieved using the scheme in section II. This leads to

$$|\phi\rangle_{910} = [|0\rangle_9(|0\rangle_{10} - |1\rangle_{10}) + |1\rangle_9(|0\rangle_{10} \oplus 1 - |1\rangle_{10} \oplus 1)]/2, \quad (8)$$

Then we perform single-qubit operation on ensemble 9 using a Raman-type pulse

$$|0\rangle_9 \rightarrow |1\rangle_9, \quad (9a)$$

$$|1\rangle_9 \rightarrow -|0\rangle_9. \quad (9b)$$

and a C-NOT transformation on the two collective 9, 10 lead Eq. (8) to

$$|\phi\rangle_{910} = [|1\rangle_9(|0\rangle_{10} \oplus 1 - |1\rangle_{10} \oplus 1) - |0\rangle_9(|0\rangle_{10} \oplus 1 - |1\rangle_{10} \oplus 1)]/2, \quad (10)$$

then another single-qubit operation

$$|0\rangle_9 \rightarrow -|1\rangle_9, \quad (11a)$$

$$|1\rangle_9 \rightarrow |0\rangle_9. \quad (11b)$$

lead the state of system to

$$|\phi\rangle_{910} = (-|0\rangle_9 - |1\rangle_9)(|0\rangle_{10} - |1\rangle_{10})/2. \quad (12)$$

For the case of performing  $U_{f3}$ , we only need a C-NOT transformation on the two collective as Eq. (10), then the system becomes

$$|\phi\rangle_{910} = (|0\rangle_9 - |1\rangle_9)(|0\rangle_{10} - |1\rangle_{10})/2. \quad (13)$$

For the case of performing  $U_{f4}$ , we first perform a single-qubit transformation of Eq. (9), and then we perform a C-NOT operation of Eq. (8). Finally, we perform a single-qubit transformation of Eq. (11). This leads to

$$|\phi\rangle_{910} = (-|0\rangle_9 + |1\rangle_9)(|0\rangle_{10} - |1\rangle_{10})/2 \quad (14)$$

Obviously we can realize the unitary transformation  $U_f$  by the above method, the state of system can be expressed as

$$|\phi\rangle_{910} = [(-1)^{f(0)}|0\rangle_9 + (-1)^{f(1)}|1\rangle_9](|0\rangle_{10} - |1\rangle_{10})/2. \quad (15)$$

This state of the two collective 9 and 10 has the same form as the Eq. (5). Then we perform a Hadamard transformation on the collective 9 by a single-qubit operation, which has been mentioned in section II. Finally we detect the state of collective 9. If the state is  $h_9^+|vac\rangle_9$ , the function  $f(x)$  is constant. Otherwise,  $f(x)$  is balanced function. Now, we have realized the Deutsch-Jozsa algorithm perfectly. Obviously a measurement is sufficient to distinguish  $f(x)$  between constant and balanced function.

#### IV. CONCLUSIONS

In summary, we present a physical scheme for implementing the Deutsch-Jozsa algorithm with atomic ensem-

bles, and the basic system is a large number of identical alkali metal atoms. The Bell-basis measurement and C-NOT gate are very important in quantum information processing and quantum computation, thus the schemes with atomic ensembles for realizing them are proposed firstly. Then we apply Bell-basis measurements and C-NOT transformations into the process of realizing Deutsch-Jozsa algorithm. The above schemes have some special advantages compared with the schemes of quantum information process by the control of single particles. For instance, it has inherent fault tolerance to the realistic noise and imperfections of entanglement generation, collectively enhanced coupling to light because of special atomic configuration and many-atom interference effects. Although the Deutsch-Jozsa algorithm is a simple example of general quantum algorithms, it is very necessary to achieve quantum computer. Further more, it would be an important step toward more complex quantum computation. The scheme involves laser manipulations of atomic ensembles, beam splitters, and single-photon detections with moderate efficiencies, which are all within the current experimental technology.

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